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Mass transfer in the spiral flow of a pseudoplastic liquid

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Abstract—The rate of mass transfer in the spiral flow of pseudoplastic liquids is experimentally investigated. The results are correlated with the usual dimensionless parameters calculated on the basis of the mean apparent viscosity value. The rate of mass transfer in the flow with Taylor vortices can be described by a correlation formulated for Newtonian liquids. This conclusion is also valid for heat transfer in the spiral flow of pseudoplastic liquids.

1. INTRODUCTION

SPIRAL flow, considered in this work, is a superposition of two basic flows in an annular gap—Poiseuille flow due to the axial pressure gradient and Couette flow caused by rotation of the inner cylinder. This flow system has many industrial applications, e.g. cooling of rotating electrical machinery [1], dynamic filtration and classification on a cylindrical surface of separation [2], the modification of the structure of alloys in the field of high rates of strain [3], the lubrication of plain bearings, etc. Applications of spiral flow in engineering of chemical reactors are studied extensively [4], including an electrochemical reactor with a rotating cylinder electrode [5]. This flow system may be very advantageous, particularly in processes involving flows of high viscous pseudoplastic liquids, due to the possibility of obtaining large rates of strain and a high intensity of secondary vortex flow, independent of the velocity of the nett flow [6].

The problem of mass transfer in a spiral flow is of great significance for the sake of application of this flow system in engineering of chemical reactors. This problem has been considered thoroughly in the case of Newtonian fluids [7–9]. With reference to non-Newtonian liquids, Poiseuille flow [10], Couette [11] and spiral flow [12] have been taken into account.

The main purpose of the present work is to determine the rate of mass transfer in the spiral flow of pseudoplastic liquids. The mass transfer at the surface of an inner cylinder is considered. Two types of spiral flow are investigated—laminar and laminar with Taylor vortices in the range of small Reynolds numbers (referred to axial flow); $Re < 200$. The results of measurements have been correlated with generalized dimensionless numbers, enabling comparison with a correlation formulated for the case of Newtonian liquids. It must be outlined, that the method of description of transport processes in flows of pseudoplastic liquids, applied in previous works (heat transfer in spiral flow [6], mass transfer in Couette flow [11]) are not adequate to make generalizations of the results obtained.

2. ANALYSIS OF THE PROBLEM

It is well known that a spiral flow retains its strictly laminar form in a range of sufficiently small angular velocities of rotation of the inner cylinder ω_i. Above a certain critical value ω_c, rotational flow loses stability which results in the creation of secondary flow (Taylor vortices) [1]. It has been shown [13] that the stability limit of the spiral flow of pseudoplastic liquids is described in a form of functional dependence of the modified critical Taylor number $Ta_c \cdot F_M$ on the Reynolds number; both dimensionless numbers have been defined with the aid of the average apparent viscosity, for the conditions of the main flow (Fig. 1)

$$\bar{\mu} = (1/d)\int_{R_1}^{R_2} \mu_a(r)\,dr \tag{1a}$$

where for power-law liquids

$$\mu_a = K\left[\left(r\frac{d\omega}{dr}\right)^2 + \left(\frac{dV}{dr}\right)^2\right]^{(1-n)/2}. \tag{1b}$$

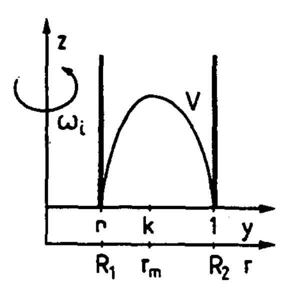

FIG. 1. Coordinate system.

NOMENCLATURE

- A parameter in equation (5) [—]
- a parameter in the dependency function of the consistency coefficient on temperature [N s^n m^{-2}]
- B dimensionless parameters of the stress field, $\pi P R_2^3/M_0$ [—]
- b parameter in the dependency function of the consistency coefficient on temperature [K^{-1}]
- C_i molar concentration of reacting ions [kmol m^{-3}]
- C_p specific heat of liquid [J kg^{-1} K^{-1}]
- D diffusion coefficient [m^2 s^{-1}]
- d gap width, (R_2-R_1) [m]
- F_0 Faraday constant [C kmol^{-1}]
- $f(\ldots)$ dimensionless function, $(B^2(y-k^2/y)^2+1/y^4)^p$
- i_c limit current density [A m^{-2}]
- K consistency coefficient [N s^n m^{-2}]
- K_0, K_w consistency coefficient values at the bulk and wall temperature, respectively
- k dimensionless parameters corresponding to the location of the maximum axial velocity (Fig. 1) [—]
- k_c mass transfer coefficient [m s^{-1}]
- L length of the mass transfer zone [m]
- M_0 torque per unit length of cylinder [N]
- Nu Nusselt number, $2\alpha d/\lambda$ [—]
- n flow index [—]
- p parameter, $(1/(2n)-1/2)$ [—]
- P axial pressure difference per unit length [N m^{-3}]
- Pr Prandtl number, $\bar{\mu}C_p/\lambda$ [—]
- Pr_G Prandtl number defined in ref. [6], $(C_pK/\lambda)\cdot(V_m^2+U_i^2)^{(n-1)/2}/(2d)^{n-1}$ [—]
- Re Reynolds number, $2dV_m\rho/\bar{\mu}$ [—]
- Re_G Reynolds number defined in ref. [6], $(2d)^n V_m^{2-n}\rho/K$
- R_1, R_2 inner and outer radius of the annular gap [m]
- r radial coordinate [m]
- Sc Schmidt number, $\bar{\mu}/(\rho D)$ [—]
- Sh Sherwood number, $2k_c d/D$ [—]
- T temperature [K]
- Ta Taylor number, $(\omega_i d^{3/2} R_1^{1/2}\rho)/\bar{\mu}$
- Ta_c critical Taylor number
- Ta_G Taylor number defined in ref. [6], $(d/R_1)(\rho d^n)U_i^{2-n}/K$
- U_i tangential velocity at the surface of the inner cylinder [m s^{-1}]
- V axial velocity [m s^{-1}]
- V_m mean axial velocity [m s^{-1}]
- y dimensionless radial coordinate, r/R_2 [—]
- z_i electrical charge involved in the electrochemical reaction.

Greek symbols

- α heat transfer coefficient [W m^{-2} K^{-1}]
- β parameter of nonisothermality of flow in equations (7) and (8), $b\psi d/\lambda$
- η radius ratio, (R_1/R_2)
- λ thermal conductivity of liquid [W m^{-1} K^{-1}]
- $\bar{\mu}$ mean apparent viscosity (equation (1)) [N s m^{-2}]
- μ_a apparent viscosity (local)
- ϕ parameter in equation (4), $(dV/dr)_{R_1}/4V_m$
- ψ heat flux [W m^{-2}]
- ω angular velocity (local) [rad s^{-1}]
- ω_c critical angular velocity of the inner cylinder
- ω_i angular velocity of the inner cylinder.

In laminar spiral flow a velocity distribution is described by the following equation [14]:

$$V(y)/V_m = (1-\eta^2)\int_\eta^y (y'-k^2/y')f(y',B,k)\,dy' \Big/ \int_\eta^1 y^2(y-k^2/y)f(y,B,k)\,dy \quad (2)$$

$$\omega(y)/\omega_i = 1-\int_\eta^y 1/y'^3 f(y',B,k)\,dy' \Big/ \int_\eta^1 1/y^3 f(y,B,k)\,dy \quad (3)$$

where k and B are dimensionless parameters which are functions of the velocity ratio V_m/U_i; $f(y,B,k)$ the dimensionless function of the apparent viscosity distribution.

In this range of flow, the rate of mass transfer can be described by the Leveque correlation [10]

$$Sh = 1.614(Re\,Sc\,2d/L\phi)^{1/3} \quad (4)$$

where

$$\phi = \left(\frac{dV}{dr}\right)_{R_1} \Big/ 4V_m.$$

The parameter ϕ is independent of the angular velocity in the spiral flow of Newtonian liquids. In the case of pseudoplastic liquids both components of flow are conjugated by a viscosity term, which results in the dependence of ϕ on the velocity ratio V_m/U_i. This

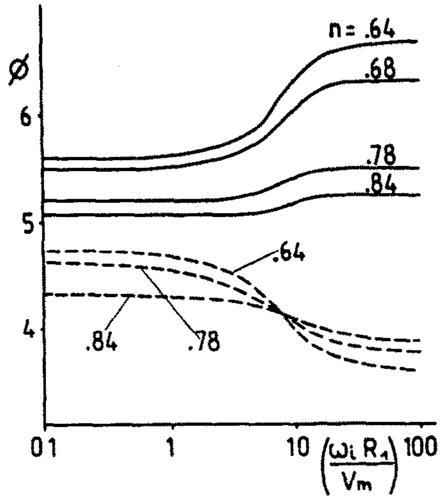

FIG. 2. Dependence of ϕ ($= (dV/dr)/4V_m$) on the velocity ratio in laminar spiral flow of power law liquid (——, inner cylinder; – – –, outer cylinder).

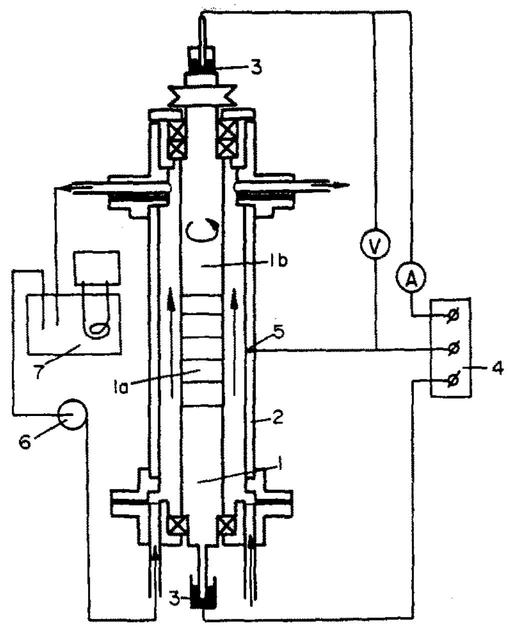

FIG. 3. Diagram of the apparatus. Test section: 1, inner cylinder; 1a, cathode rings; 1b, anode; 2, outer cylinder; 3, mercury contact; 4, potentiostat; 5, reference electrode; 6, pump; 7, reservoir.

parameter can be calculated from the velocity distribution (equations (2) and (3)).

For example, results of the calculation of ϕ for $\eta = 0.66$ are shown in Fig. 2. Rotational motion involves a certain increase in the mass transfer coefficient at the surface of the inner cylinder; results for the outer cylinder display an opposite tendency (dashed line).

In the case of the spiral flow with vortices ($Ta > Ta_c$), an increase of the angular velocity implies an increase of the secondary flow intensity, which results in a considerable growth of the rate of mass transfer. Legrand and Coeuret [8] have formulated the following correlation, for the case of Newtonian liquids, valid in the range $25 < Re < 300$:

$$Sh = A\, Ta^{1/2}\, Sc^{1/3} \qquad (5)$$

where the constant A depends on the radius ratio η.

3. EXPERIMENTS

The rate of mass transfer is determined with the aid of the electrochemical method [15]. The reaction of the cathodic reduction of ferricyanide ions at a nickel electrode is used. When the reaction occurs under concentration–polarization control, the following relationship between the limit current density and the mass transfer coefficient is obeyed:

$$i_c = k_c z_i F_0 C_i. \qquad (6)$$

The potential of the cathode is measured with reference to a platinum electrode submerged in a flowing solution.

A diagram of the apparatus is shown in Fig. 3. It consists of four basic parts—the test section, the reservoir connected to a thermostat, the circulation pump and the electrical system for electrochemical measurements.

The main part of the test section is the inner bored rotating cylinder, which consists of three segments—the inlet segment 200 mm in length (made of stainless steel), the cathode divided into four isolated nickel rings (each 18 mm long) and the outlet segment, serving simultaneously as an anode. The thickness of the insulation between the cathode rings is smaller than 0.1 mm. The anode is made of a nickelized steel shaft. The inner cylinder is positioned with bearings placed at each end of the cylinder. The bearings are fixed between two stainless steel collars. The outer cylinder, made of Pyrex, is fixed between the collars. The apparatus is equipped with three interchangeable cylinders which have inside radii of 46.4, 50, and 60 mm, respectively. The cathode and the anode are connected to a stabilized power supply (potentiostat) through mercury contacts placed at each end of the inner cylinder. In the present investigation four cathode rings connected together have been used (thus the length of the mass transfer zone is 72 mm).

Water solutions of carboxymethylcellulose are used to carry out experiments (the polymer concentration is between 0.45 and 0.8% w/w).

For the sake of the electrochemical method used, the solutions contain 0.2 mol l^{-1} sodium bicarbonate (as a supporting electrolyte), 0.01 mol l^{-1} potassium ferrocyanide and 0.005 mol l^{-1} potassium ferricyanide.

The polymeric solutions displayed non-Newtonian behaviour; their rheological characteristics may be described by the power law equation. The values of the flow index varied in the range 0.64–0.95.

The polymeric solutions were prepared 24 h before the experiments. The samples of potassium ferri- and ferrocyanide were added on the day of the experiment and the solutions obtained were treated with nitrogen. Before measurements were taken, preliminary circulation of the solution was initiated to stabilize the rheological properties of the system.

4. RESULTS

Spiral flows of pseudoplastic liquids (laminar as well as laminar with Taylor vortices) are characterized by variability of a viscosity value in the flow field. In order to correlate results of mass transfer measurements in a dimensionless form, an appropriate reference viscosity value should be defined. This problem is especially important in the case of flow with vortices.

It can be considered, that parameters characterizing a secondary flow velocity are much less relevant, than the components of the main flow velocity [7]. In the case of pseudoplastic liquids, a viscosity distribution is conditioned mainly by the main flow. Taking into account the fact that the intensity of the secondary flow depends on viscous forces acting over the whole gap, leads to the conclusion that the mean viscosity value (equation (1)) may be used in correlating results of measurements of mass transfer in the flow of pseudoplastic liquids.

For a fixed axial flow velocity, the dependence of the limit current value on the angular velocity ω_i has been determined.

The results of measurements have been correlated with the mean viscosity value, determined for parameters of the main flow—V_m and ω_i. Values of the diffusion coefficient of ferricyanide ions in CMC solutions determined by Shulman *et al.* [16] are used.

The results concerning a spiral flow without as well as with vortices are shown in Figs. 4 (the wider gap) and 5 (two narrower gaps), respectively.

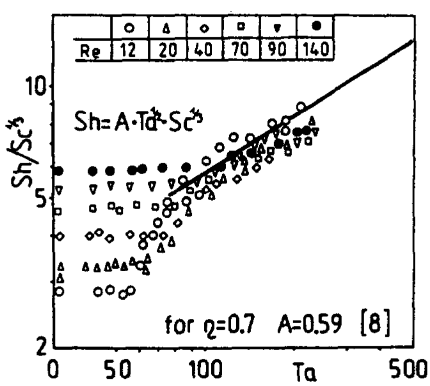

FIG. 4. Results of mass transfer measurements (wide gap $\eta = 0.66$).

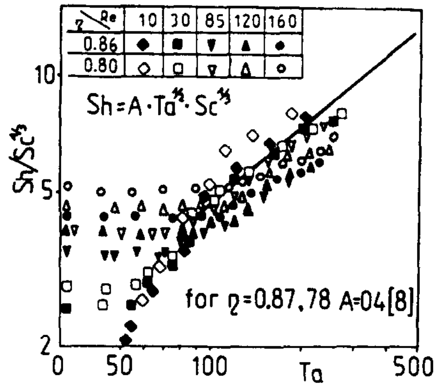

FIG. 5. Results of mass transfer measurements (narrow gaps $\eta = 0.8$ and 0.86).

In the case of the flow with vortices, the mass transfer coefficient increases with Ta and is practically independent of Reynolds number. The results presented are in agreement with the experimental correlation (5) formulated for Newtonian liquid flows, the average deviation of the experimental points from the correlation is within 12% for all gaps used in the work.

In the case of the pseudoplastic liquids, applying a mean apparent viscosity value (equation (1)) enables the rate of mass transfer to be described in a spiral flow with vortices by the correlation, formulated for flows of Newtonian liquids (equation (5)). This conclusion is consistent with other cases of convective mass transfer in non-Newtonian liquids; correlating the rate of mass transfer with the aid of an apparent viscosity calculated for a properly defined nominal value of the rate of shear leads to an agreement with correlations formulated for Newtonian liquids (for example, flows in a tube, in a packed bed, etc.).

In the range of spiral flow without vortices, the rate of mass transfer is described by equation (4). For a constant Peclet number ($= Re\ Sc$), ϕ being a function of n and U_i/V_m, comprises the influence of the rotational motion (expressed by Taylor number) and the pseudoplasticity of liquid (flow index n) on mass transfer in this range of flow. The analysis of variability of ϕ within the ranges of parameters considered in this work (η, n, U_i/V_m) proves that even in the case of the widest gap ($\eta = 0.66$) the values of the mass transfer coefficient differ only slightly from those predicted for Newtonian liquids; the difference is less than 9% (for comparison see Fig. 2). As can be concluded from Figs. 4 and 5, the mass transfer coefficient is independent of the Taylor number in the range of spiral flow without vortices. Rotational motion does not affect the rate of mass transfer, which is consistent with the theoretical predictions mentioned above.

5. COMPARISON WITH HEAT TRANSFER MEASUREMENTS

The formulated method of description of mass transfer in a spiral flow of pseudoplastic liquids has

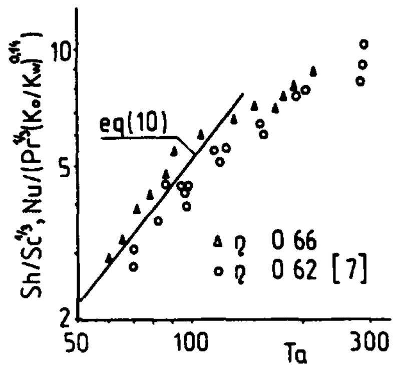

FIG. 6. Comparison of heat and mass transfer measurement in spiral flow with vortices: heat transfer in power law liquid (equation (10)), mass transfer in power law liquid (Δ) and Newtonian liquid (O).

been used in the analysis of a problem of heat transfer. Nouar *et al.* [6] carried out investigations of the heat transfer from a surface of the outer cylinder on the spiral flow of a pseudoplastic liquid (3% w/w CMC, $n = 0.5$). Applying certain modified forms of dimensionless numbers (marked Pr_G, Re_G), they obtained the following experimental correlations which include the influence of variation of the consistency K with temperature (parameter β):

(a) spiral flow without vortices

$$Nu = 1.24\beta^{0.177}\, Pr_G^{1/3}\, Re_G^{0.17}\, Ta_G^{0.08}; \qquad (7)$$

(b) spiral flow with vortices

$$Nu = 0.036\beta^{0.177}\, Pr_G^{1/3}\, Ta_G^{1.026}. \qquad (8)$$

The above correlations have been obtained in the following ranges of parameters: $Re < 15$; $Ta < 120$ (both numbers defined with the mean viscosity value); $\eta = 0.61$. The dependence of the consistency K on temperature has been described by the relation

$$K = a \exp(-bT).$$

Introducing the numbers Re, Ta, Pr defined by the mean viscosity value (equation (1)) and substituting β by the term of the Sieder–Tate type $(K_0/K_w)^{0.14}$ [17] leads to the modified forms of correlations (7) and (8)

$$\text{(a)} \qquad Nu = 1.5(Re\,Pr)^{1/3}(K_0/K_w)^{0.14}; \qquad (9)$$

$$\text{(b)} \qquad Nu = 0.017Ta^{1.25}\, Pr^{1/3}(K_0/K_w)^{0.14}. \qquad (10)$$

Applying the analogy between heat and mass transfer and taking values of ϕ, $(2d/L)$ calculated for the experiments in ref. [6] enable equation (4) to be transformed into the form [17]

$$Nu = 1.614(Re\,Pr)^{1/3}(K_0/K_w)^{0.14}. \qquad (11)$$

The results of Nouar *et al.*'s experiments for spiral flow without vortices exhibit a very good agreement with Leveque's modified correlation (compare equations (9) and (11)).

In Fig. 6, results of heat transfer measurements in a spiral flow with vortices, described by equation (10), have been compared with those concerning the mass transfer in a spiral flow of Newtonian liquids, obtained by Kataoka *et al.* [7] in a similar range of parameters—$\eta = 0.62$ and $Re < 12$. A good agreement can be concluded for $Ta < 120$, which corresponds to the range of validity of equation (10). The results of the present work obtained for $\eta = 0.66$ and $Re = 12$ are shown in Fig. 6 also; they are consistent with the results of Kataoka *et al.* in the whole range of Ta; a very good agreement with equation (10) in the initial range of Ta must be outlined.

6. CONCLUSIONS

In the case of pseudoplastic liquids, applying the mean apparent viscosity value (equation (1)) enables one to describe the rate of mass transfer in accordance with the results for Newtonian liquids. This conclusion is also valid for the heat transfer in the spiral flow of pseudoplastic liquids. The rate of mass transfer in the flow with Taylor vortices is described by correlation (5) formulated for Newtonian liquids. In the range of parameters, considered in the present work ($n > 0.6$, $\eta > 0.6$), the rotational motion has a negligible influence on the rate of mass transfer in the laminar flow without vortices.

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APPENDIX A. HYDRODYNAMICS OF LAMINAR SPIRAL FLOW OF POWER LAW LIQUID

The velocity of a spiral flow consists of two non-zero components—angular and axial components. The distributions of the components of the shear stress are known to be (cylindrical coordinates on Fig. 1)

$$\tau_{rz} = M(y-k^2/y) \tag{A1}$$

$$\tau_{r\theta} = M/y^2. \tag{A2}$$

The components of the rate of the strain tensor take the form

$$\dot{\gamma}_{rz} = 1/R_2 \frac{dV}{dy} \tag{A3}$$

$$\dot{\gamma}_{r\theta} = y\frac{d\omega}{dy} \tag{A4}$$

where M, N are parameter of the stress field corresponding to the axial pressure gradient and the applied torque, respectively. The viscosity function for a power liquid is

$$\mu = K^{1/n}\left(\sum_{i,j}\tau_{ij}\tau_{ji}\right)^{(n-1)/2n}. \tag{A5}$$

After substituting equations (A1) and (A2) into equation (A5) the following viscosity distribution is obtained:

$$\mu(y) = K^{1/n}/N^{(1-1/n)}\cdot f(y,B,k) \tag{A6}$$

where the function $f(..)$ is defined as

$$f(y,B,k) - (B^2(y-k^2/y)^2+1/y^4)^p \tag{A7}$$

and

$$B = M/N, \quad p = 1/2n-1/2.$$

Combining equations (A1)–(A4) and (A6), applying the definition of viscosity ($\mu = \tau_{ij}/\dot{\gamma}_{ij}$) and integrating the equations obtained for the gradients of the velocity components results in dimensionless profiles of the angular and axial velocities

$$V(y)/V_m = (1-\eta^2)\int_\eta^y (y'-k^2/y')\cdot f(y',B,k)\,dy' \bigg/ \int_\eta^1 y^2(y^2-k^2/y)\cdot f(y,B,k)\,dy \tag{A8}$$

$$\omega(y)/\omega_i = 1-\int_\eta^y 1/y^3 f(y',B,k)\,dy' \bigg/ \int_\eta^1 1/y^3 f(y,B,k)\,dy. \tag{A9}$$

The following expression for the mean axial velocity V_m has been used in equation (A8):

$$V_m = \frac{MR_2}{(1-\eta^2)K^{1/n}}N^{(1/n-1)}\int_\eta^1 y^2(y-k^2/y)f(y,B,k)\,dy. \tag{A10}$$

The profiles of both velocities can be determined from equations (A8) and (A9) by numerical integration. The unknown values of k and B can be determined from the following set of two algebraic equations:

$$k^2 = \int_\eta^1 y\cdot f(y,B,k)\,dy \bigg/ \int_\eta^1 1/y\cdot f(y,B,k)\,dy \tag{A11}$$

$$B = V_m/U_i(1-\eta^2)\int_\eta^1 1/y^3 f(y,B,k)\,dy \bigg/ \int_\eta^1 y^2(y-k^2/y)f(y,B,k)\,dy. \tag{A12}$$

APPENDIX B. TRANSFORMATIONS OF CORRELATIONS (7) AND (8)

The dimensionless numbers Re_G, Ta_G, Pr_G used in ref. [6] can be expressed by the numbers Re, Ta, Pr (calculated on the basis of the mean apparent viscosity value)

$$Re_G = Re\cdot 2^{n-1}\frac{\bar{\mu}}{K(U_i/d)^{n-1}}(U_i/V_m)^{n-1} \tag{B1}$$

$$Ta_G = Ta\cdot(d/R_1)^{1/2}\cdot(\bar{\mu}/K(U_i/d)^{n-1}) \tag{B2}$$

$$Pr_G = C_p\frac{K(U_i^2+V_m^2)^{(n-1)/2}}{\lambda(2d)^{n-1}} \approx Pr\cdot 2^{(1-n)}\frac{K(U_i/d)^{n-1}}{\bar{\mu}}. \tag{B3}$$

The last equation is justified by the fact that in the experiments analysed the velocity ratio (U_i/V_m) was greater than 3. Using the definition of the Nusselt number results in the expressions for β

$$\beta = \frac{b\psi d}{\lambda} = Nu\frac{b(T_w-T_b)}{2}. \tag{B4}$$

(a) Correlation (7)—the flow without vortices

The product $Re_G^{0.17}\,Ta_G^{0.08}$ in equation (7) is transformed into the following form:

$$Re_G^{0.17}\,Ta_G^{0.08} = Re^{0.17}(U_i/V_m)^{((n-1)0.17)}\,Ta^{0.08} \times\left(\frac{\bar{\mu}}{K(U/d)^{n-1}}\right)^{-0.08}(d/R_1). \tag{B5}$$

Putting $n = 0.5$ and $(d/R_1) = 0.62$ results in

$$Re_G^{0.17}\,Ta_G^{0.08} \approx Re^{0.25}\left(\frac{\bar{\mu}}{K(U_i/d)^{n-1}}\right)^{0.25}. \tag{B6}$$

Applying equations (B6) and (B4) leads to the modified form of correlation (7)

$$Nu = 1.23Pr^{0.4}\,Re^{0.3}\left(\frac{b(T_w-T_b)}{2}\right)^{0.2}\left(\frac{\bar{\mu}}{K(U_i/d)^{n-1}}\right)^{-0.1}. \tag{B7}$$

The assumed dependence of the consistency index on temperature leads to the following expression for the Sieder–Tate term $((K_b/K_w)^{0.14})$:

$$(K_b/K_w)^{0.14} = \exp(0.14b(T_w-T_b)). \tag{B8}$$

It has been found that, in the range of temperature differences $(T_w-T_b) = 10$–100°C, the approximate relationship can be formulated as

$$(K_b/K_w)^{0.14} \approx 1.5(b(T_w-T_b)/2)^{0.2}. \tag{B9}$$

This enables equation (B7) to be transformed into the final form given by equation (9); it has been assumed that

$$(\bar{\mu}/K(U_i/d)^{n-1})^{-0.1} \approx 1 \quad \text{(B10)}$$

$$Pr^{0.07}(1200-5200) \approx 1.7. \quad \text{(B11)}$$

(b) Correlation (8)—the flow with vortices

The experiments for this flow have been carried out in the range $60 < Ta < 120$, and the velocity ratio value (U_i/V_m) was greater than 20. This enables the approximate viscosity distribution function to be formulated

$$\dot{\gamma}_{\theta r} \gg \dot{\gamma}_{Zr} \rightarrow \mu_a = K(\dot{\gamma}_{\theta r}^2)^{(n-1)/2}. \quad \text{(B12)}$$

Equation (B12) leads to the mean viscosity value $\bar{\mu}$

$$\bar{\mu} = K\frac{1-\eta^{[(2/n)-1]}}{1-\eta}\,\frac{n}{2-n}\left[\frac{\frac{2}{n}\omega_i}{\eta^{-2/n}-1}\right]^{n-1}. \quad \text{(B13)}$$

In this case, the dimensionless numbers Pr_G and Ta_G can be expressed in the following form:

$$Pr_G \approx 1.4Pr \quad \text{(B14)}$$

$$Ta_G \approx 0.8Ta. \quad \text{(B15)}$$

The expressions given above have been inserted into equation (8); coefficient β has been substituted by the term $(K_b/K_w)^{0.14}$ according to the method described earlier, which results in the final form given by equation (10).

TRANSFERT DE MASSE DANS UN ECOULEMENT SPIRALE DE FLUIDE PSEUDOPLASTIQUE

Résumé—On étudie expérimentalement le flux de masse transféré dans un écoulement spiralé de liquides pseudoplastiques. Les résultats sont corrélés avec les paramètres adimensionnels usuels calculés sur la base d'une viscosité apparente moyenne. Le flux de masse transféré dans l'écoulement avec tourbillons de Taylor peut être décrit par une formule adaptée aux liquides newtoniens. Cette conclusion est aussi valable pour le transfert thermique dans l'écoulement spiralé des liquides pseudoplastiques.

STOFFTRANSPORT IN SPIRALFÖRMIGEN STRÖMUNGEN PSEUDOPLASTISCHER FLÜSSIGKEITEN

Zusammenfassung—Der Stofftransport in spiralförmigen Strömungen pseudoplastischer Flüssigkeiten wird experimentell bestimmt. Die Meßergebnisse werden mit den gewöhnlicherweise verwendeten dimensionslosen Parametern korreliert, basierend auf der mittleren Viskosität. Der Stofftransport in Strömungen mit auftretenden Taylor-Wirbeln kann durch eine Korrelation, wie sie auch für Newtonsche Flüssigkeiten gilt, beschrieben werden. Diese Folgerung hat ebenso Gültigkeit für den Wärmetransport in spiralförmigen Strömungen pseudoplastischer Flüssigkeiten.

МАССОПЕРЕНОС В ЗАКРУЧЕННОМ ПОТОКЕ ПСЕВДОПЛАСТИЧЕСКОЙ ЖИДКОСТИ

Аннотация—Численно исследуется интенсивность массопереноса в закрученном потоке псевдопластической жидкости. Результаты обобщаются на основе обычных безразмерных параметров, рассчитанных по среднему значению кажущейся вязкости. Скорость массопереноса в потоке с вихрями Тэйлора можно описать соотношением, сформулированным для ньютоновских жидкостей. Этот вывод справедлив и для теплопереноса в закрученном потоке псевдопластических жидкостей.